

# The properties of the surface of Minkowski space, which determine the type of its Grassmann image

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The properties of the surface of the Euclidean space, which determine the values or boundaries of the values of the curvature of the Grassmann manifolds along planes tangential to the Grassmann image of the surface have been investigated [1], [2], [3]. The results of solving the similar problems for the surfaces of the Minkowski space depend on the type of their Grassmann image. In this paper, the properties of the surfaces of different types that determine the type of their Grassmann image are investigated.

In Minkowski space  ${}^1R_4$  (with metric  $ds^2 = -dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2$ ), the submanifolds of the space-like and the time-like planes of the Grassman manifold  $PG(2, 4)$  are pseudo-Riemannian four-dimensional manifolds of the pseudo-Euclidean space  ${}^3R_6$ . The tangent space for each of these submanifolds is the space with a signature metric  $(- - ++)$  [4].

The Grassmann image of the space-like (time-like) two-dimensional surface of the space  ${}^1R_4$  is a two-dimensional submanifold of the manifold of time-like (space-like) planes [4]. The induced metric of the Grassmann image may be sign-definite, sign-indefinite or degenerated, and, hence, the Grassmann image can be a two-dimensional space-like, time-like or isotropic surface.

**Proposition 1.** *If the time-like surface  $V^2 \subset {}^1R_4$  has a flat normal connection, then its Grassmann image is a time-like surface.*

**Proposition 2.** *If the time-like (space-like) surface  $V^2 \subset {}^1R_4$  is minimal (maximum), then its Grassmann image is a space-like surface.*

**Proposition 3.** *If the surface  $V^2 \subset {}^1R_4$  is a hypersurface of a some three-dimensional subspace, then the type of its Grassmann image coincides with the type of the surface.*

## REFERENCES

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